On the Spectra of Turbulent Fluids at Large k

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The turbulence spectra of continuous fluids at large k are discussed. A basic difference in behaviour appears between conservative and dissipative systems. Only in the latter case can one eliminate higher order ultraviolet divergences.

In previous work on the equilibrium statistics of the Korteweg de Vries equation (see [1, 2]) and other 1-d non-integrable but dispersive equations [3], it was possible to prove that the space correlation function has an exponential decay like $e^{-\alpha|x|}$. This was the result of the simultaneous presence of nonlinearity and dispersion in the equation. Otherwise the correlation function would have a $\delta(x)$ behaviour.

The k-spectrum being the Fourier transform of the correlation function, the behaviour at large k of the spectrum corresponds to the behaviour at small x of the correlation function. The $e^{-\alpha|x|}$ behaviour suppresses the crude ultraviolet divergence in the energy spectrum and leads to a Lorenz shape as k^{-2} (see [4]). This crude divergence occurs in case of a $\delta(x)$ behaviour of the correlation function, which corresponds to equipartition. Since the derivatives of $e^{-\alpha|x|}$ are not continuous or even defined, divergences can appear in the spectra of the derivatives of the fluid variables. This situation seems to be unavoidable for conservative continuous systems.

Dissipative fluids have a fundamentally different behaviour. It has been proved for 2-d Navier-Stokes equations that the time-independent space correlation function is smooth for all x (see [5]). This has been used recently to estimate the large k behaviour of the energy spectrum, which turns out to be exponentially decaying (see [6]). In [6] the smoothness (C^∞) of the correlation function has been assumed for the 3-d case without proof. This assumption seems plausible, however, for dissipative fluids which usually have attractors with finite (see [7]) Hausdorff dimension, though proofs in the 3-d case are missing. This suggests that they behave like a system with a finite number of degrees of freedom for which no ultraviolet-like divergences of any type can occur.

In the dissipative case, the divergences are avoided whatever the ensemble average is, as long as ergodicity is assumed, while in the conservative case the kind of averaging is important because suitable weighting among the infinite number of degrees of freedom penalizes, for instance, the large k (see [2]). The penalization is, however, suitable for certain quantities but not for others, especially those containing higher derivatives, so it cannot be guaranteed that all ultraviolet divergences are suppressed. Viscous-like dissipation seems to be the way to eliminate completely the ultraviolet problem.

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